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NICOLAVS TARTAGLIA GEOMET

THE MATHEMATICS TEACHER

Volume XXIII

No. 6

Edited by William David Reeve

Tenth Year Mathematics Outline*

By W. D. REEVE AND HIS STUDENTS

Teachers College, Columbia University, New York, N.Y.

A GREAT MANY teachers of geometry are recommending a combination course of plane and solid geometry, one year in length. The attitude of the National Committee favors such a course, as will be seen in these quotations from its recent report.

"It is felt that the work in plane geometry gives enough training in logical demonstration to warrant a shifting of emphasis in the work of the solid geometry away from this aspect of the subject, and in the direction of developing greater facility in visualizing spatial relations and figures. . . . . Some schools will find it possible and desirable to introduce the elementary notions of solid geometry in connection with related ideas of plane geometry."

". . . . it would seem wise to have at least some of the work in solid geometry come as early as possible in the mathematical course, preferably not later than the beginning of the eleventh school year."

The following course is outlined to carry out this idea. If the student has had intuitional geometry in the junior high school, he has the necessary preliminary work in the matter of vocabulary, geometric ideas and construction. He also should have had numerical trigo-

* A great many students at Teachers College have helped to develop this outline. It is impossible to give all of their names here as the work has extended over a period of three or four years.

nometry in the ninth year. If further, he has had a unit of demonstrative geometry of six or eight weeks in the ninth year, as is now being advocated by some leaders—there is no doubt but that there will be plenty of time to teach a combined course in one year. The Fifth Year Book of the National Council of Teachers of Mathematics discusses this unit of demonstrative unit for the ninth year and gives two such sample units in detail.

This outline assumes that the student has had intuitional geometry and numerical trigonometry. Since the demonstrative unit is taught in so few schools and since the subject may vary for those schools that do have it we are not assuming it has been taught. If it has, been taught, this course can be considerably shortened. If neither intuitive geometry or numerical trigonometry has been taught it will have to be condensed.

Since we consider our main objective to be, to give students the meaning of proof, that is, a method of thinking, the original exercises become the important part of the work. For this reason, our committee recommends that, we teach the solving of original exercises by analysis. This will give our students "a way to discover truths." An article[1] in the Fifth Year Book discusses this method. Many of the theorems listed in this outline may be made original exercises.

We have not listed exercises. The new books have many interesting ones. Such texts as Strader and Rhoades, Stone and Mallory, R. R. Smith, Hassler, Haertter, Mirick-Newell and Harper, Clark and Otis, and Nyberg emphasize original work. Breslich's "The Teaching of Mathematics" will also be helpful.

Outline of Tenth Year Mathematics

Axioms—Those found in any book of recognized authority.

List of Postulates and Fundamental Theorems

This list is to be accepted as assumptions or from intuitional or experimental proofs and explanations.

1. Two points determine a line.
2. A line segment may be produced any desired length.
3. Two straight lines can intersect in only one point.
4. A straight line is the shortest distance between two points.
5. The perpendicular is the shortest distance from a point to a line.

[1] Schlauch, W. S., Fifth Yearbook National Council Teachers of Mathematics.

6. One and only one perpendicular can be drawn through a given point to a line.

7. Through a given point not on a straight line, one straight line and only one can be drawn parallel to the given line. (Parallel Postulate)

8. If two lines are perpendicular to two intersecting lines, they are not parallel.

9. Lines drawn from the vertices of angles of a triangle and lying within the triangle must intersect.

10. If two lines are cut by a transversal and the corresponding angles are equal the lines are parallel.

11. Converse of 10. (To be preceded by this definition of parallels, "Parallel lines are those that have the same amount of turning from an initial line." See Reeve, General Mathematics Book II).

12. All right angles are equal.

13. All straight angles are equal.

14. If two adjacent angles are supplemental, their exterior sides form straight line.

15. If two adjacent angles have their exterior sides in a straight line, they are supplementary.

16. Complements of equal angles are equal.

17. Supplements of equal angles are equal.

18. Vertical angles are equal.

19. If two angles are both equal and supplementary each is a right angle.

20. From a given center with a given radius, one and only one circle can be described in a plane.

21. All radii of a circle or equal circles are equal.

22. Diameters of a circle or equal circles are equal.

23. A straight line can intersect a circle in at most two points.

24. In the same circle or equal circles, equal central angles intercept equal arcs.

25. Converse of 24.

26. A central angle is measured by its intercepted arc.

27. If the distance from a point to the center of a circle is greater than the radius, the point lies outside of the circle and conversely.

28. If less, the point lies within the circle and conversely.

29. If two triangles have two sides and the included angle of

the one equal, respectively to two sides and the included angle of the other, the triangles are congruent.

30. If two triangles have two angles and the included side of one, equal respectively to two angles and the included side of the other, the triangles are congruent.

31. If two triangles have three sides of the one equal, respectively, to three sides of the other the triangles are congruent.

32. The area of a rectangle is equal to the product of its base and altitude.

33. The circumferences of two circles are to each other as their radii.

34. The area of a circle equals one half of the product of the circumference and radius.[1]

35. The intersection of two planes is a straight line.

36. If a line is perpendicular to a given plane, every plane which contains this line is perpendicular to the given plane.

37. If each of two intersecting planes is perpendicular to a third plane, their line of intersection is perpendicular to that plane.

38. The volume of a rectangular solid equals the product of the area of its base and altitude.

Theorems to be Proved and Ideas to be Developed[2]

Congruent Triangles—Perpendiculars

Plane Geometry

1. Proof for familiar constructions:
   (a) Triangle from three given lines.

[1] It is pretty generally agreed, that proof by the theorem of limits is beyond the high school pupil. An understanding is established by explanation of the terms constant, variable and limit, but proof using the theorem of limits is not given by many authorities. Stone-Mallory establishes this postulate, "Any theorem which has been proved true regarding a regular polygon and which does not depend on the number of sides of the polygon is true for the circle." Other authors not using the theorem of limits are:

Reeve, General Mathematics Book II
Strader-Rhoads, Plane Geometry
R. R. Smith, Beginners Geometry
Bernard, Plane Geometry
Avery, Plane Geometry
Hassler, Plane Geometry

[2] Unless otherwise indicated all the following are theorems to be proved. If crowded for time, the starred exercises may be accepted as postulates or omitted.

(b) Perpendicular bisector of a line.
(c) Perpendicular from point off line to line.
(d) Perpendicular at point on line to line.
(e) Bisecting an angle.
(f) Copying an angle.

2. In an isosceles triangle, the angles opposite the equal sides are equal.

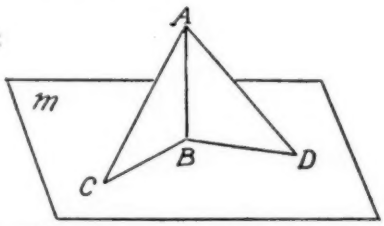
3. Many exercises on congruent triangles.

4. Review of perpendicular line ideas.

SOLID GEOMETRY

1. Definition of plane.
2. Discussion to bring out four ways of fixing a plane.
3. Definition of line perpendicular to plane.
4. Exercises on perpendicular to plane, such as

Given: AB $\perp$ to plane m
With C and D equally distant from B
To Prove: AC=AD



also practical problems, as inaccessible distances. (See Reeve, General Mathematics Book II.)

For a general discussion of planes and perpendiculars to planes, See Clark-Otis Solid Geometry.

5. Through a given point in a plane, one and only one line can be drawn perpendicular to the plane.

6. Through a given point not in a plane, one and only one line can be drawn perpendicular to the plane.

Note: Theorems 5 and 6 offer an easy use of indirect proof. See Professor Upton's article in the Fifth Year Book for a splendid discussion of indirect proof.

7. A line perpendicular to each of two intersecting planes, is perpendicular to the plane of these lines.

Note: This theorem is sometimes called the key theorem of solid geometry. If a student understands this proof and gets the space perception with it, he is very likely to have no trouble with solid

geometry. As prerequisite he should have from plane geometry, these two theorems.

(a) Any point on the mid perpendicular to a line is equally distant from the ends of the line.

(b) Two points each equally distant from the ends of a line, fix the mid perpendicular to that line.

Parallels

PLANE GEOMETRY

1. If two lines are cut by a transversal and the alternate interior angles are equal the lines are parallel.
2. Converse of 1.
3. If a line is perpendicular to one of two parallel lines it is perpendicular to the other also.
4. Two lines perpendicular to the same line are parallel.
5. Two lines parallel to the same line are parallel.
6. Construct a line parallel to a given line.

SOLID GEOMETRY

1. Definition of parallel planes.
2. Write a list of theorems for solid corresponding to those of plane geometry. See Nyberg, Solid Geometry, p. 307.
3. If two parallel planes are cut by a third plane, the lines of intersection are parallel.
4. Definition and discussion of dihedral angle.
5. Discussion and definition of plane angle of dihedral.
6. Definition and discussion of right dihedral angle, acute angle and the like.

Triangles and Angles

PLANE GEOMETRY

1. The sum of the angles of a triangle equals 180°
2. The exterior angle of a triangle equals the sum of the two opposite interior angles.
3. If two angles of a triangle are equal respectively to angles of another triangle, the third angles are likewise equal.
4. If two angles of a triangle are equal, the sides opposite are equal and the triangle is isosceles.
5. If two right triangles have hypotenuse and side of one equal respectively to hypotenuse and side of the other, the triangles are congruent.

6. Construct a right triangle having given a side and the hypotenuse.

7. Construct a right triangle having the hypotenuse and an acute angle, given.

8. If two angles have their sides parallel, they are equal or supplementary.

9. If two angles have their sides perpendicular, they are equal or supplementary.

Parallelograms

PLANE GEOMETRY

1. Either diagonal of a parallelogram divides it into two congruent triangles.

2. Opposite sides of a parallelogram are equal.

3. Opposite angles of a parallelogram are equal.

4. Diagonals of a parallelogram bisect each other.

5. If a quadrilateral has its opposite sides equal, it is a parallelogram.

6. If a quadrilateral has one set of sides equal and parallel it is a parallelogram.

SOLID GEOMETRY

1. Discussion of properties of faces of parallelopipeds, and prisms.

2. Exercises on planes cutting through prism and parallelopipeds.

3. If two angles not in the same plane have their sides parallel and extending in the same direction, they are equal.

4. Plane angles of a dihedral are equal.

Special Theorems on Triangles and Polygons

PLANE GEOMETRY

1. In a 30°-60° right triangle, the hypotenuse is double the side opposite the 30° angle. (Note: This theorem deserves especial emphasis because of its use in higher mathematics.)

2. A line bisecting two sides of a triangle is parallel to the third side and equal to half of it.

3. If two angles are unequal, the sides opposite are unequal and the greater side is opposite the greater angle.

4. If two sides of a triangle are unequal, the angles opposite are unequal and the greater angle lies opposite the greater side.

5. The sum of the interior angles of a polygon equals (n-2) 180°.

SOLID GEOMETRY (optional)

1. Illustration and discussion of trihedral and polyhedral angles.
2. By intuitive proof, The sum of the face angles of a polyhedral angle is less than 360°.
3. Discussion of the five regular convex polyhedrons.

Locus

PLANE GEOMETRY

1. From recent examination results, it is apparent that locus is not being taught as successfully as we would wish. A more thorough understanding of the meaning of locus, before any formal proving, may help. Notice Stone-Mallory Plane Geometry, Smith-Reeve-Morse Plane Geometry Test Books, Clark-Otis Plane Geometry, R. R. Smith Beginner's Geometry or Haertter's Plane Geometry, for preliminary exercises on this topic.
2. By illustrations and problems develop the seven important locus ideas.
3. The mid-perpendicular to a line is the locus of all points equidistant from the ends of the line.
4. The bisector of an angle is the locus of all points equally distant from the sides.
*5. The perpendicular bisectors of the sides of a triangle meet in a point.
*6. The bisectors of the angles of a triangle meet in a point.
*7. The altiudes of a triangle meet in a point.
*8. The medians of a triangle meet in a point that is two-thirds of the distance from the vertices of the triangle to the base. (If the proof of this is omitted, the fact should be illustrated and accepted for the higher mathematics.)

SOLID GEOMETRY

1. State locus ideas in terms of solid geometry. See Clark-Otis's or Nyberg's Solid Geometry.
2. Any point in the plane bisecting the dihedral angle is equally distant from the faces of the angle.

Circles

PLANE GEOMETRY

1. In the same circle or equal circles, equal arcs have equal chords.

2. Converse of 1.

3. A diameter perpendicular to a chord bisects the chord and the arc subtended by it.

4. In the same circle or equal circles, equal chords are equidistant from the center.

5. Converse of 4.

6. A line perpendicular to a radius at its outer extremity is tangent to the circle.

7. Converse of 6.

8. Tangents drawn to a circle from a point outside the circle are equal.

9. Construct a tangent to a circle at a point on the circle.

10. Construct a tangent to a circle from a point outside the circle.

11. To inscribe a regular hexagon in a circle.

12. To inscribe a square in a circle.

SOLID GEOMETRY

1. Definition of a sphere.

2. Discussion to bring out that 4 points determine a sphere.

3. Discussion of circles on a sphere and application to earth's surface.

4. Discussion of distance on a sphere—(navigation and aviation).

5. A plane cuts a sphere in a circle.

Angle Measurement and the Circle

PLANE GEOMETRY

1. An inscribed angle is measured by half the intercepted arc.

2. An angle formed by two chords intersecting within the circle is measured by half the sum of the intercepted arcs.

3. An angle formed by tangent and chord is measured by half the intercepted arc.

4. An angle formed by two tangents or two secants or a tangent and a secant, drawn from a point outside the circle to the circle is measured by half the difference of the intercepted arcs.

5. Arcs of a circle included between parallels are equal.

SOLID GEOMETRY

1. Definition of a spherical angle and its measurement.

Area

PLANE GEOMETRY

1. The area of a parallelogram equals base times altitude.
2. The area of a triangle equals half the base times altitude.
3. Area of a trapezoid equals half the sum of the bases times the altitude.

SOLID GEOMETRY

1. Develop formulas for lateral area of prism, pyramid and frustum of pyramid.
2. Develop formulas for volume of a prism and a pyramid.

Ratio and Proportion

PLANE GEOMETRY

1. Understanding of ratio and its use.
2. Understanding of proportion, its laws and uses.
3. A line parallel to the base of a triangle divides sides proportionally.
4. Converse of 3.
5. If a series of parallels are cut by a transversal, the corresponding segments are proportional.
6. If a series of parallels cut off equal segments on one transversal, they cut off equal segments on any transversal.
*7. The bisector of the interior angle of a triangle divides the opposite side into segments proportional to the adjacent sides of the angle.
8. Construction: To divide a line into equal parts.
9. If a triangle has an angle of one triangle equal to an angle of another triangle, the areas are to each other as the products of the sides including the angles.

SOLID GEOMETRY

1. Parallel planes cut off proportional segments on transversals.
2. A plane parallel to the base of a pyramid divided the slant height, the altitude, and the lateral edges proportionally.

Similar Polygons

PLANE GEOMETRY

1. Mutually equiangler triangles are similar.

2. Two triangles are similar if an angle of one equals an angle of another and the including sides are proportional.

3. If two triangles have their sides respectively proportional, they are similar.

4. Corresponding triangles of similar polygons are similar.

5. Converse of 5.

6. Perimeters of similar polygons have the same ratio as corresponding sides.

7. Areas of similar triangles have the same ratio as the squares of corresponding sides.

*8. Areas of polygons have the same ratio as the square of corresponding sides.

SOLID GEOMETRY

1. If a pyramid is cut by a plane parallel to the base, the section is similar to the base.

2. What would be the corresponding solid geometry theorem for 6 and 7 above?

3. What would you think might be the comparison of volumes of similar polyhedrons?

Proportional Lines in Circles

PLANE GEOMETRY

1. If two chords intersect within a circle the product of segments of one equals the product of the segments of the other.

2. If from point without a circle a tangent and secant are drawn, the tangent is a mean proportional between the secant and the exterior portion.

SOLID GEOMETRY

1. Exercises on sphere based on the above theorem, which gives the formula $d = \sqrt{3h/2}$ in which h = height in feet above sea level, and d = distance in miles from which a light can be seen.

Special Theorems On Right Triangles

PLANE GEOMETRY

1. The altitude upon the hypotenuse of the right triangle forms two right triangles similar to the given triangle and each other.

2. The perpendicular from the right angle of a right triangle to the hypotenuse is a mean between the segments of the hypotenuse.

*3. If the perpendicular to the hypotenuse of a right triangle is

drawn, each side is a mean between the hypotenuse and the segment adjacent to that side.

4. The square on the hypotenuse is equal to the sum of the squares of the other two sides.

5. Construction: The mean proportional between two given lines.

Regular Polygons

PLANE GEOMETRY

1. A circle may be inscribed in any regular polygon.

2. A circle may be circumscribed about any regular polygon.

3. The area of a regular polygon equals half the product of the perimeter and the apothem.

4. Regular polygons of the same number of sides are similar.

5. Construction: Any regular polygon, inscribed and circumscribed.

6. By trigonometry develop these formulas:

a. Perimeter of the regular inscribed polygon $= n \sin\left(\frac{360^\circ}{2n}\right) d$

b. Perimeter of the regular circumscribed polygon = n tangent $\left(\frac{360^\circ}{2n}\right)d$

(n = number of sides, d = diameter)

(See Reeve, General Mathematics Book II for a complete discussion.)

SOLID GEOMETRY

1. Exercises on finding slant height, altitude and apothem in pyramid.

Limits, Constants, Variables

PLANE GEOMETRY

1. As a preparation for measurement of circle, an understanding of meaning and use of variable, constant and limit is necessary. Notice Mirick-Newell-Harper Plane Geometry, p. 154 or Stone-Mallory p. 440. Give special emphasis to circumference of a circle as a limit which the perimeters of the inscribed and circumscribed circles approach as the sides are indefinitely increased.

2. Having postulated $C/C' = r/r'$ and Area equals half the circumference times radius derive $C/D = \pi$.

3. Determine the value of π by using the formulas for perimeters of regular polygons as noted under regular polygons above.

By approximation methods and using many place tables, this can be found as accurately as we wish.

SOLID GEOMETRY

1. Extend limit idea to apply to pyramids for the cones and to prisms for the cylinders.
2. Develop formulas for lateral area of cone and cylinder, also volumes of cylinders and cones.
3. Develop formulas for area of sphere and volume.

LIST OF SUGGESTED EXERCISES FROM WHICH ORIGINAL WORK MAY BE CHOSEN

This list includes facts, commonly appearing in text books as theorems. If presented by analysis, they can be proved and so increase the number of geometric ideas.

Involving Congruent Triangles

1. Corresponding altitudes of congruent triangles are equal.
2. Corresponding medians of congruent triangles are equal.
3. Bisectors of corresponding angles of congruent triangles are equal.
4. Bisectors of the base angles of an isosceles triangle are equal.
5. The median, altitude or bisector of the vertex angle of an isosceles triangle divides the triangle into congruent right triangles.

Parallel Lines

1. If two lines are parallel and are cut by transversal, the two interior angles on the same side of the transversal are supplemental.
2. Converse of 1.
3. Same for exterior angles.
4. If a line is parallel to one of two parallels it is parallel to the other also.

Parallelograms and Polygons

1. Two consecutive angles of a parallelogram are supplementary.
2. Diagonals of a rectangle are equal.
3. Diagonals of a rhombus are perpendicular.
4. The long diagonal of a kite divides it into two congruent triangles.
5. The short diagonal of a kite divides it into two isosceles triangles.

6. The base angles of an isosceles trapezoid are equal.
7. The diagonals of an isosceles trapezoid are equal.
8. The exterior angles of a polygon are equal to four right angles.
9. Each exterior angle of a regular polygon equals $360°/n$.
10. Each interior angle of a regular polygon $= [(n—2)\ 180°]/n$.
11. If the diagonals of a quadrilateral bisect each other, the equadrilateral is a parallelogram.
12. Parallels between parallels are equal.
13. Perpendiculars between parallels are equal.

Triangles and Quadrilaterals

1. All angles of an equilateral triangle are equal.
2. Converse.
3. The median, the altitude, the bisector of the angles drawn from the vertex angle are identical.
4. The midpoint of the hypotenuse of a right triangle is equidistant from all vertices.
5. The midjoin of a triangle is parallel to the base.
6. The median of a trapezoid is parallel to the base and equals half the sum of the bases.

Circles

1. In the same circle or equal circles, chords equally distant from the center are unequal.
2. Converse of 1.
3. A line perpendicular to a tangent at the point of tangency passes through the center of the circle.
4. The line of centers of two tangent circles passes through the point of tangency.

Areas

1. Two triangles are equal if they have equal bases and equal altitudes.
2. The median of a triangle divides it into two equal triangles.
3. The diagonals of a parallelogram divide it into four equal triangles.
4. The area of a rhombus equals half the product of the diagonals.
5. Area of an equilateral triangle $= (a^2\sqrt{3})/4$.

Similar Polygons

1. Corresponding altitudes of similar triangles are same ratio as any two corresponding sides.
2. Same for corresponding medians, or bisectors of the angles.
3. The perimeters of corresponding triangles are to each other as any two corresponding lines.
4. Same for polygons.

National Council of Teachers of Mathematics YEARBOOKS

The first Yearbook on "A General Survey of Progress in the last Twenty-five Years" is out of print. The second on "Curriculum Problems in Teaching Mathematics" may be secured for $1.25. The third on "Selected Topics in Teaching Mathematics," the fourth on "Significant Changes and Trends in the Teaching of Mathematics Throughout the World Since 1910," and the Fifth Yearbook on "The Teaching of Geometry" each may be obtained for $1.75 (bound volumes), from the **Bureau of Publications, Teachers College, 525 West 120th Street, New York City.**

Problems in the Teaching of Mathematics

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THE PURPOSE OF THIS study is to present a systematic classification of the mathematical literature which relates to the problems in the teaching of secondary school mathematics and which has been written during the last three decades. This literature consists of a large number and variety of articles published in numerous educational and mathematical journals, a smaller number of special studies dealing with specific problems, some books on the psychology and teaching of the subject, a few books of a general nature discussing the philosophical aspects of the concepts and principles of elementary mathematics, and unpublished theses. The collection and presentation of this vast material in a connected form is a much needed, though laborious and difficult, task. Its classification will be based upon three distinct, and at the same time inter-related principles: first, that there is a real need for a summary of the problems relating to the teaching of secondary school mathematics; second, that these problems should be analyzed in the light of available literature; and third, that the sources of references dealing with each problem should be brought together.

The analysis of literature dealing with the problems in the teaching of mathematics presents many difficulties. For example, it is necessary to determine a basis or criterion to be used in the evaluation of the worth of a publication toward its inclusion in the list of references. In the attempt to determine such a criterion the following questions arise. Shall a publication be included on the basis of the method employed, or of the nature of results? Is a so-called experimental study, for example, one which reports the results of administering certain tests to "equated" groups and computing the coefficients of correlations, to be ranked the same as a non-experimental article, for example, one which discusses the nature of the function concept in mathematics? Should an article presenting the opinion of the author based upon years of actual classroom experi-

ence be considered as worthy as one written by a graduate student who reports the results of a study based upon an anlysis of textbooks? Is a philosophical discussion of the pillar concepts of mathematics by an expert mathematician of sound and broad mathematical training as valuable to the teachers of mathematics as the results of an experimental study by a psychologist with meager mathematical training?

Furthermore, certain difficulties arise in the classification of the problems. A classification of the problems in the teaching of mathematics for one purpose may be entirely inadequate for another. Shall they be classified on the basis of their importance to the teacher of mathematics, or of their significance to the investigator? Shall the classification be according to the field in which the problem lies or according to the method of investigation which is used in its solution?

In mathematics, as in all other fields of research, there are four types of problems encountered: (1) the historical, (2) the experimental, (3) the survey, and (4) the philosophical, each bearing the name of the method employed in its solution. The historical method bases its conclusions on a detailed and critical study of the history of the development of the problem and of the contributions toward its solution. The experimental method employs controlled experiments, tests, or experiences to discover, develop, or verify a principle. The survey method collects information toward the solution of the problem from the existing literature on the subject, from the opinions of the authorities in the field through personal interviews, personal correspondence, and questionnaires. The philosophical, or synthetical method, uses logical exposition to establish the truth of a proposition. Each method has its own advantages and limitations and is best suited for the study of certain types of problems.

The four terms above will be used to characterize the studies which report results of research in the analysis or solution of specific problems. Obviously, any one of the four research methods may be used effectively, either singly or jointly, in the analysis of a problem, the nature of the problem determining the particular method or methods to be used. Hence, a reported study will often be characterized by more than one term, if the investigator has used more than one method of research in the analysis or solution.

A summary of problems relating to the teaching of mathematics

based upon the results of investigations alone would of necessity eliminate from consideration all but the studies which employ one, or more, of the four research methods, and would include only these in the source of references. But the second basic principle of classification of mathematical literature proposes to analyze the problems in the light of the available literature. This means that the summary will consist not only of the problems which have been actually investigated, but will include also all problems which have been considered significant enough to be discussed as such in various publications. These articles, though not research studies, will be included in the list of references on the basis of their contributions to the presentation, formulation, or analysis of the problem, and will be characterized as experimental, or subjective, as the case may be.

The numerous problems in the teaching of secondary school mathematics, with their appropriate sources of references, will be classified under six main headings: (1) problems relating to the determination of objectives in the secondary school mathematics; (2) problems relating to the organization of the subject matter; (3) problems dealing with the methods and modes to be used in teaching; (4) problems relating to evaluations of the results of teaching; (5) problems dealing with the psychological aspects of mathematics; and (6) problems relating to the philosophy of mathematics. These classes of problems constitute the general research fields within each of which are to be found the specific and definite problems for investigations.

The list of problems encountered in mathematical literature constitutes the theme of the present paper. Subsequent papers will report on each class of problems, and in particular will present the following considerations of the problems of each class:

1. Analysis of each problem in terms of its logical elements.
2. Formulation of the problem in terms of these elements.
3. Consideration of the significance and value of the problem in mathematical education.
4. Analysis of the problem in terms of its relation to, and bearing upon, the associated problems of other fields. Many fields overlap at the point where they have problems in common.
5. Analysis of the methods used in the solution of the problem, if the problem has been carried beyond the stage of formulation.
6. Suggestions of methods of solution for each problem, where the analyzed facts warrant such a course.

7. Presentation and interpretation of the results and findings in the case of those problems for which solutions have been reported.

8. A complete source of references will be listed for each problem, and a more general bibliography for each class.

The question of the functions of mathematics in secondary education has been a source of much discussion and dispute for many years. A number of problems have been presented which at the present time engage the attention of educators and investigators working in the field of objectives. A survey of mathematical literature discloses the following problems relating to the objectives of mathematics:

1. To determine the bases on which we are to decide upon the objectives in the teaching of mathematics. Such factors may be considered as interest of pupils, practical uses, and social values.
2. To evaluate and come to an agreement regarding the procedures to be followed in the determination of the objectives in the teaching of mathematics. This involves such procedures as activity analysis, survey of practical and social uses, analysis of textbooks, and recommendations of authorities and experts.
3. To state the general mathematical objectives, for example:
   a. The formation and acquisition of certain habits and attitudes.
   b. The appreciation of concepts and processes.
4. To define the relation of the general mathematical objectives to the general educational objectives, that is, to the *Cardinal Principles* of education.
5. To clarify the relation of the mathematical objectives to the objectives of other sciences, for example, the physical and social sciences.
6. To select the specific objectives of mathematics, for example:
   a. The acquisition of certain skills and mastery of mathematical processes.
   b. The acquisition of certain understandings of processes, methods, and principles.
7. How to relate the objectives of mathematics to the development of the science of mathematics.
8. To ascertain the values to be derived from mathematical instruction, for example: the vocational, social, and recreational values.
9. The determination of mathematical objectives for different levels, such as:
   a. The junior high school.
   b. The senior high school.
   c. The junior college.
10. To determine the course objectives of mathematics.
    a. The specific and general objectives of arithmetic.
    b. The specific and general objectives of algebra.
    c. The specific and general objectives of geometry.
    d. The specific and general objectives of trigonometry.

e. The specific and general objectives of analytics.
f. The specific and general objectives of the elementary calculus.

11. To determine the unit objectives for the various mathematical courses.

The problems relating to the organization of the subject matter of mathematics deal with the selection of instructional materials for the courses, and the assembling and arranging of the selected materials in convenient and appropriate units of instruction. These problems are closely related to the problems in the other five fields because the selection and arrangement of instructional materials depend upon objectives, methods of teaching, psychological considerations, and the philosophical significance attributed to the material. The formulation of the problems relating to the field of organization of the subject as revealed in the literature and listed below is based upon the principles of selection and arrangement of the material for the purposes of instruction.

1. Problems relating to the selection of the subject matter of mathematics. These problems may be grouped under the following topics:
   a. The elimination of certain topics in arithmetic, algebra, and geometry in view of the objectives of the course.
   b. The elimination, or removal to a higher level, of certain topics in arithmetic and algebra in view of their teachability.
      (1) The arithmetic of economics to be taught as a senior high school subject.
   c. The selection and inclusion of certain topics traditionally taught at a high level.
      (1) The selection and organization of algebraic work in junior high school mathematics.
      (2) The selection and organization of geometry in junior high school mathematics.
      (3) The organization of analytic geometry as a high school subject.
      (4) The introduction and organization of the calculus as a high school subject.
2. Problems relating to the method of arrangement of the subject matter of mathematics. They will be classified under the following topics:
   a. The traditional organization of mathematics.
   b. The historical organization of mathematics.
   c. The logical organization of subject matter in mathematics.
   d. The psychological arrangement of subject matter.
   e. The parallel plan of arrangement of subject matter.
   f. The spiral system of arrangement of subject matter.
   g. Arrangement of material according to the inductive method of approach.
   h. The grade placement of subject matter.

3. Problems relating to the organization of the subject matter of mathematics into courses. They will be grouped under the following headings:
   a. The organization of courses in correlated mathematics.
      (1) The organization of the newer type of arithmetic.
      (2) The correlation of arithmetic, algebra, geometry, trigonometry in junior high school mathematics.
      (3) The correlation of algebra and geometry in senior high school mathematics.
      (4) The correlation of plane and solid geometry.
      (5) The correlation of analytics with other subjects of senior high school mathematics.
      (6) The correlation of elementary calculus with other subjects of senior high school mathematics.
      (7) The survey courses in junior college mathematics.
   b. The unitary organization of subject matter.
   c. The organization courses in mathematics to be required of all pupils.
   d. The organization of the elective courses in mathematics.
   e. The organization of courses in mathematics to satisfy the college entrance requirements.
   f. The organization of mathematics needed in other school subjects.
   g. The organization of applied and vocational mathematics.
4. Problems relating to psychological considerations in the organization of subject matter of mathematics.
   a. To organize subject matter to provide for individual differences.
   b. To organize subject matter in its relation to drill material.
   c. To organize subject matter as related to remedial work.
   d. To organize subject matter in its relation to the time allotment.
   e. To organize subject matter into psychological units of instruction.
5. Problems relating to the organization of subject matter according to methods of teaching.
   a. To organize material for the laboratory method.
   b. To organize material for the individualized method of instruction.
   c. To organize material into projects.
   d. To organize material for supplementary project work.
   e. To organize material for the purpose of motivation.
   f. To organize appropriate historical materials.
6. Problems relating to modern tendencies in organization of subject matter of mathematics.
   a. To organize subject matter suitable to various types of high schools.
   b. To organize subject matter in its relation to objectives.
   c. To organize materials to conform to the types revealed through textbook analyses.

The field of methods and modes of teaching mathematics contains problems which deal with various ways of presenting, unfolding, and developing the subject matter of mathematics in the classroom, and

with their relative effectiveness. These problems are related, on the one hand, to the problems of objectives and organization, and on the other to the problems of evaluation of results. For their complete formulation and satisfactory solution, certain problems in this field are dependent upon psychological problems. Others, mostly dealing with the intrinsic values of the modes of instruction, are closely related to the problems of a philosophical nature. The problems listed below are related to methods of teaching secondary school mathematics:

1. Problems relating to the classroom procedure in mathematics, such as:
   a. Supervised study in mathematics.
   b. The recitation in mathematics.
   c. The laboratory method in mathematics.
   d. The individualized method of instruction in mathematics.
   e. The project method in mathematics.
   f. The Chicago Plan.
   g. The Dalton Plan.
   h. The Winnetka Plan.
2. Problems relating to presentation and development of the subject matter of mathematics, such as:
   a. The inductive and deductive modes.
   b. The analytic and synthetic modes.
   c. The lecture mode.
   d. The heuristic mode and the art of questioning.
3. Problems relating to psychological considerations of methods of teaching mathematics, like the following:
   a. Motivation in mathematics.
   b. Elimination of errors and difficulties of manipulation.
   c. Reading in the teaching of mathematics.
   d. The administration of drill work in mathematics.
   e. Remedial work in mathematics.
   f. Provisions for individual differences in mathematics classes.
      (1) The grouping of pupils.
      (2) The use of supplementary material.
4. Teaching problems relating to the philosophical considerations of the concepts and processes, as the following:
   a. The rationalization of various mathematical processes.
   b. The teaching of analysis.
   c. The teaching of generalizations and abstractions.
   d. The teaching of special cases and applications.
   e. The teaching of symbolism and notations.
   f. The teaching of relationships in mathematics.
   g. The teaching of mathematical laws.
   h. The teaching of fundamental concepts of mathematics.

5. Problems relating to the teaching of various mathematical subjects. They are collected under the following topics:
   a. Teaching methods relating to arithmetic.
      (1) The rationalization of arithmetical processes.
      (2) The solution of verbal problems in arithmetic.
      (3) Utilization of algebraic symbols in arithmetic.
      (4) The teaching of statistical methods.
   b. Teaching methods relating to algebra.
      (1) The teaching of the operations of algebra.
      (2) The teaching of alegraic representation and algebraic notations.
      (3) The teaching of algebraic functions.
      (4) The teaching of graphic methods.
      (5) The teaching of algebraic applications, such as logarithms and the slide rule.
      (6) The teaching of equations in algebra.
   c. Teaching methods relating to plane geometry.
      (1) The teaching of demonstration in geometry.
      (2) The teaching of construction problems in geometry.
      (3) The teaching of the original problems in geometry.
      (4) The teaching of functionality in geometry.
   d. Teaching methods relating to trigonometry.
      (1) The teaching of the trigonometric functions.
      (2) The teaching of the trigonometric applications.
   e. Teaching methods relating to solid geometry.
      (1) The use of models in the teaching of solid geometry.
      (2) The teaching of drawing and projection in solid geometry.
      (3) The teaching of functionality in solid geometry.
   f. Teaching methods relating to elementary analytic geometry.
      (1) The teaching of the representation in analytic geometry.
      (2) The teaching of functionality in analytic geometry.
      (3) The teaching of the functions represented in analytic geometry.
   g. Teaching methods relating to elementary calculus.
      (1) The teaching of the operations of the calculus.
      (2) The teaching of the applications of the calculus.
      (3) The teaching of functionality in the calculus.

Problems relating to evaluation of results.—Problems relating to the field of evaluation of the results of mathematical instruction are mensurational in character. In order to find means of measuring results of instruction it must be assumed that such results can be measured. Of course the measured results in this field, like all measured results, are approximate, but may be significant if interpreted carefully. The problems are of four different types: (1) to determine "learning units" in terms of which the results of instruction are to be evaluated; (2) to construct tests to be used as instruments for evaluating

the results of instruction; (3) to determine efficient methods of administering tests; and (4) to interpret and apply the test results. The problems listed below often combine two or more of these types as their relevant elements, and are concerned mostly with the last three types—the first type seems to be grouped with the problems relating to the determination of objectives of mathematics.

1. Problems relating to the determination of learning units in mathematics.
   a. To determine the unit objectives.
      (1) The immediate objectives.
      (2) The contingent objectives.
   b. To determine the course objectives.
      (1) The specific objectives.
      (2) The general objectives.
2. Problems relating to the construction of types of tests in mathematics.
   a. To construct unit tests.
      (1) The pretest or preview.
      (2) The presentation test.
      (3) The assimilation test.
      (4) The mastery test.
   b. To construct course tests.
      (1) The period test (semester).
      (2) The final examination.
      (3) The testing of specific objectives.
      (4) The testing of general objectives.
   c. To construct tests to measure mathematical ability.
      (1) The prognostic tests to measure innate abilities.
      (2) The achievement tests to measure the acquired abilities.
         (a) The power tests.
         (b) The objective tests.
         (c) The standardized tests and scales.
   d. To construct diagnostic tests.
   e. To construct practice tests.
3. Problems relating to the construction of forms of tests in mathematics.
   a. The essay type test.
   b. The true and false test.
   c. The multiple choice test.
   d. The completion test.
   e. The matching test.
4. Problems relating to the administration of tests.
   a. To determine the use of self-administering tests.
   b. To determine how to use the practice tests.
   c. To determine how to use the diagnostic tests.
   d. To determine how to use the standardized tests.
   e. To determine how to use the prognostic tests.
   f. To determine how to use the achievement tests.

5. Problems relating to the interpretation and application of the test results.
   a. To determine the validity of test results.
   b. To determine the reliability of test results.
   c. To use test results in the determination of objectives.
   d. To use test results in curriculum construction.
   e. To determine the value of tests in making instruction purposeful.
   f. To determine the use of tests in remedial work.
   g. To determine the use of test results in evaluating pupil's work.
      (1) The numerical marks.
      (2) The literal marks.
6. Problems relating to the value of college entrance examinations.

Problems relating to psychological considerations.—In the fifth class, which has been called the psychological aspects of mathematics, are grouped all problems relating to psychological considerations in the determination of objectives, the organization of subject matter, the teaching methods and procedures, and the evaluation of the results of instruction. The educational psychologist has made definite contributions toward the improvement of mathematical instruction, through the applications of the psychology of learning and the psychology of individual differences. The following problems may be classified as relating to the psychological aspects of mathematics.

1. Problems relating to the determination of mathematical abilities.
   a. Ability to study mathematical subjects.
      (1) The measure of mental ability.
      (2) The significance of individual differences.
   b. The nature of mathematical abilities.
      (1) Mathematical skills.
      (2) Mathematical attitudes.
      (3) Mathematical appreciations.
   c. The significance of mathematical abilities in reflective thinking.
   d. The relation between mathematical abilities and general intelligence.
   e. The relation between mathematical abilities and mental efficiency.
   f. The relation between mathematical abilities and sex.
   g. The relation between mathematical abilities and inheritance.
   h. The relation between mathematical abilities and success in mathematical subjects.
      (1) Success in algebra.
      (2) Success in geometry.
   i. The relation between mathematical abilities and success in other subjects.
      (1) Success in physical sciences.
      (2) Success in social sciences.

2. Problems relating to psychological considerations in the determination of objectives of mathematics.
   a. Analysis of values of mathematical instruction.
      (1) Habit formation.
      (2) Acquisition of skills.
      (3) Obtaining information.
      (4) Formation of attitudes.
      (5) Acquisition of appreciations.
   b. To determine the transfer of training in mathematics.
      (1) Retention of mathematical training.
      (2) Relation between transfer and method of teaching.
      (3) Greater transfer in some mathematical abilities than in others.
   c. To determine the nature of disciplinary values of mathematics.
3. Problems relating to psychological considerations in the organization of subject matter of mathematics.
   a. To organize materials psychologically.
      (1) The introduction of new topics.
      (2) The development of difficult topics.
      (3) The meanings and definitions.
      (4) Psychological sequence of algebraic topics.
   b. To correlate materials.
      (1) Topics rather than processes.
      (2) Minimize interference.
4. Problems relating to psychological considerations in the teaching of mathematics.
   a. To use inductive methods.
      (1) Large number of observations.
      (2) Observation of likenesses and differences.
      (3) Discover generalizations.
   b. To apply the laws of learning.
      (1) Learning by doing.
      (2) Meaning with experience.
      (3) Forming correct habits.
      (4) Bond analysis.
      (5) Drill.
         (a) Good drill materials.
         (b) Methods of drill.
         (c) Distribution of drill.
         (d) Fatigue in drill.
      (6) Real learning situations.
      (7) Speed and accuracy.
      (8) Relative difficulty of processes and concepts.
   c. To motivate teaching.
      (1) Pupil interests.
         (a) Puzzles and recreations.
         (b) Nature objects.
         (c) Practical applications.

d. To consider individual differences.
   (1) Nature of individual differences.
   (2) Amount of individual differences.
   (3) Grouping according to ability.
e. To apply remedial treatment.
   (1) Analysis of errors in arithmetic.
   (2) Analysis of errors in algebra.
   (3) Analysis of reading in mathematics.
f. To apply psychological procedures in classroom.
   (1) Supervised study.
   (2) The assignment.
   (3) The art of questioning.

5. Problems relating to psychological consideration in the evaluation of results of mathematical instruction.
   a. To measure mathematical abilities.
      (1) Skills.
      (2) Understandings.
      (3) Appreciations.
   b. To measure habits formed by mathematical training.
      (1) Discriminating between true and false.
      (2) Discriminating between assumption and demonstration.
      (3) Discrimination between accuracy and approximations.

Problems relating to the philosophy of mathematics.—This class of problems is concerned with the elucidation of the meaning of mathematical concepts; the correct interpretation of mathematical symbolism and notations; the rationalization of the processes and operations of mathematics; the explanation of the true significance of mathematical representations and relationships; the determination of the levels at which the difficult mathematical doctrines should be treated; the presentation of the historical development of mathematical ideas; the coordination of mathematical abstractions, generalizations, and applications; and the evaluation of the contributions of mathematics to civilization and the functions of mathematics in the general scheme of education. The foregoing problems are related to the problems of the determination of objectives by analyzing any proposed list of objectives in the light of the genuine functions of mathematics. They are also related to the field of organization of the subject matter by furnishing guiding principles for the unfolding and development of mathematical concepts, processes, and principles. They are related to the problems in the field of methods and modes of teaching by analyzing the characteristics, the functions, and the proper technique of the various methods and modes used in the teach-

ing of mathematics. They are related to the problems of the evaluation of the results of teaching by their concern in the determination of genuine learnings which are to be used as units of measure of mathematical instruction. Finally the problems in this field are directly associated with those in the field of the psychology of mathematics by interpreting the psychological aspects of mathematics in the light of the philosophy of mathematics.

The problems listed below are suggested by their presentation and discussion in mathematical literature:

1. Problems of a philosophical nature relating to the interpretation of the functions of mathematics.
   a. To evaluate and interpret correctly functions of mathematics in the advancement of civilization.
      (1) Historical considerations showing the relation of mathematics to civilization in the past.
      (2) Analysis of the relations of mathematics to the present day civilization.
   b. To evaluate and interpret correctly functions of mathematics in the sciences.
      (1) Relation of mathematics to physical sciences.
      (2) Relation of mathematics to social sciences.
      (3) Relation of mathematics to economic sciences.
      (4) Relation of mathematics to political sciences.
   c. To evaluate and interpret correctly functions of mathematics in the industry and commerce.
      (1) Relations of mathematics to the industrial phenomena and problems.
      (2) Relations of mathematics to the commercial enterprises.
   d. To evaluate and interpret correctly functions of mathematics in general scheme of education.
      (1) Relation of mathematics to industrial education.
      (2) Relation of mathematics to humanistic education.
   e. To evaluate and interpret correctly functions of mathematics in secondary education.
      (1) Relation of functions of mathematics to functions of other school subjects in secondary education.
      (2) The unique functions of mathematics in secondary education.
   f. To interpret the nature of objectives of secondary school mathematics in terms of the genuine functions of mathematics.
      (1) Analysis of the objectives generally accepted at the present time.
      (2) Suggestions for the determination of more acceptable objectives.
2. Problems of a philosophical nature relating to the interpretation of the meaning, nature, and significance of mathematical ideas, methods, and applications to improve the selection and organization of secondary school mathematics.

a. To determine what mathematical ideas and methods may be proved, without a great deal of difficulty, in secondary schools.
b. To determine what mathematical ideas and methods may be presented in secondary schools without proof.
   (1) The geometric principles in junior high school.
   (2) The principles of analytic geometry and the calculus in senior high school.
c. To determine a feasible sequence for the presentation of mathematical ideas and methods in each course that will facilitate their mastery. For example,
   (1) Sequence of theorems in geometry.
   (2) Sequence of processes and principles in algebra.
d. To determine a feasible sequence of courses in secondary school which will enable the realization of the functions of mathematics.
e. To determine the nature and amount of historical material to be presented in mathematics courses.
   (1) To aid in the proper development of the subject.
   (2) To aid in the motivation of the subject.
f. To determine the application of mathematics which should receive the greatest emphasis in secondary school.
g. To determine the coordination of mathematical ideas and methods to facilitate their mastery, e.g.,
   (1) Geometric interpretation of algebraic operations.
   (2) Algebraic representation of geometric relationships.
   (3) The methods of analytic geometry in the interpretation of algebraic functions.
   (4) The methods of the calculus in the interpretation of algebraic functions.
   (5) The methods of the calculus in the interpretation of geometric concepts.

3. Problems of a philosophical nature relating to the interpretation of the meaning, nature, and significance of mathematical ideas, methods, and applications to improve the presentations and development of secondary school mathematics.
   a. To present advanced mathematical ideas and methods for the extension of the teacher's mathematical knowledge. For example:
      (1) The foundations of geometry.
      (2) Non-Euclidean geometries.
      (3) Theory of numbers.
      (4) Theory of equations.
      (5) Theory of groups.
      (6) The number system.
      (7) Theory of functions of variables.
   b. To interpret correctly the meaning of basic mathematical ideas and methods treated in secondary schools, such as:
      (1) Mathematical elements and undefined terms.
      (2) Mathematical definitions and concepts.

(3) Mathematical assumptions and postulations.
(4) Mathematical relationships and propositions.
(5) Mathematical operations.
(6) Mathematical notations and symbols.
(7) Mathematical processes and methods.
(8) Mathematical theories.
(9) Mathematical truths.
(10) Mathematical generalizations and laws.

c. To determine the significance and applications of mathematical ideas and methods which might be used to supplement classroom instruction.
(1) Ideas and methods suited for supplementary project work.
(2) Ideas and methods suited for lectures and special presentations.
(3) Ideas and methods suited for presentation by mathematical clubs.

d. To determine the proper levels for the presentation of new ideas and methods.

e. To determine the proper levels for the rationalization of old ideas and methods.

4. Problems of a philosophical nature relating to the analysis of methods and modes used in the teaching of mathematics.
a. To distinguish the characteristics of the various methods of the development and presentation of mathematical subjects, e.g.:
(1) The analytic and the synthetic methods.
(2) The inductive and deductive methods.
(3) The Socratic and the heuristic methods.
(4) The laboratory and the lecture methods.

b. To determine the specific functions of each method in the development and rationalization of mathematical ideas and processes.

c. To present the historical development of each method.

5. Problems of a philosophical nature relating to the determination of types of learnings in mathematics in terms of mathematical ideas, methods, and applications.
a. To acquire understanding of the nature of, and skill in the use of, symbolic representation.

b. To acquire understanding and appreciation of the quantitative method of analysis.

c. To acquire understanding of the nature of, and ability to discern between, approximate and exact relationships.

d. To acquire correct habits of functional thinking.

e. To acquire understanding of the nature of, and ability to discern between, provisional and absolute truths.

f. To acquire appreciation of, and ability to use, logical rigour.

g. To acquire understanding and appreciation of universal laws.

h. To acquire understanding of the nature of generalization and appreciation of their applications.

Experimental Work and Special Classes in Mathematics in the High Schools of New York City

AT THE REQUEST of numerous visitors to the various city schools the following list was compiled by the Syllabus Committee of the Association of Chairmen of Mathematics Departments of New York City. The members were allowed more than a month to reply to the request for information about their departments. It is, therefore, assumed that the schools not mentioned in this list are carrying on only the usual high school courses.

1. *Alexander Hamilton High School,* Ralph P. Bliss, Chairman.

(a) From among those electing elementary algebra for the first time, a number are chosen who give evidence of being good students. These are placed in what is called a Rapid Advance Class which completes all of the topics of elementary algebra in one term with five recitations a week. None of the topics taken by the regular classes are omitted, but much of the drill work must be curtailed. Those who attain a grade of 75 per cent in the final examination are promoted into intermediate algebra. The others are placed in the second term of the regular work. All students must take algebra for one full school year. The class is intended to help the brighter students. Repeaters are not allowed to register. It has been found inadvisable to allow students to drop algebra at the end of the first term even if they receive a grade of 100 per cent in the examination. The successful ones receive credit for three terms of work in one year.

(b) In the teaching of the required topics of the elementary algebra syllabus, the question of the best order from the psychological standpoint has been considered. One aspect of the problem has been neglected, namely, the matter of the summer recess intervening between two halves of the year's work. The following order of topics has been found feasible:

First term:

Introduction to algebra
Algebraic numbers

The four fundamental operations
Linear equations
Ratio, proportion, variation
Graphic representation
Systems of linear equations

Second term:

Special products and factors
Fractions and equations containing fractions
Square root, radicals and exponents
Quadratric equations
Trigonomtery of the right triangle

The desirable feature of this list of topics is the increasing difficulty of the topics. The simpler ones are given first. The student is not upset with factoring, algebraic fractions, and the more complicated work until he is well started.

(c) A mathematics study class is maintained for those who wish to join it and who have shown special ability in mathematics. This is called a study class, but it is a work class in a very real sense of the word. There is no home work, but the teacher assigned to this class is free to introduce any topics in algebra geometry or more advanced parts of mathematics that she finds suitable to the work at hand. Individuals with special aptitude are encouraged to continue with mathematics.

2. *Bushwick High School,* Aaron Freilich, Chairman.

An X Y Z experiment with first year pupils is now in operation the purpose of which is to exact from each pupil an honest response commensurate with the pupil's native ability. Provision is made in all assignments and examinations for three levels of intelligence usually found in classes not grouped according to ability. These levels are called X (slow 65% – 69%), Y (average 70% – 89%) and Z (bright 90% – 100%). Mimeographed lesson plan and assignment sheets definitely outlining the required classroom and home work for the entire term are placed in the hands of each pupil. All work is rated X, Y, or Z or F (failure) and records of this work are kept from day to day. Pupils are constantly urged and encouraged to rise into higher levels.

3. *Girls Commercial High School,* Margaret C. Byrne, Chairman.

(a) General mathematics for two terms is required of all pupils taking the Industrial Arts Course. This includes units of intuitive

geometry, geometrical construction including dynamic symmetry, demonstrative geometry, algebra including the graph and arithmetic.

(b) Commercial mathematics for one term is required of all second term pupils taking the Commercial Course. This includes the graph, the formula, the equation, computation especially in percentage, emphasis on relations between numbers, rounding off large numbers and approximately results. The pupils are grouped on the basis of a Group Intelligence Test into A, B, C groups.

4. *George Washington High School,* Joseph B. Orleans, Chairman.

(a) The best pupils in geometry are grouped separately in the second half of the year and are given a somewhat enriched course.

(b) The pupils who have done creditable work in elementary algebra and in plane geometry are assigned to a special class, if it meets their approval, in which one year's work is done in one term. Up to this term the work consisted of intermediate and advanced algebra. This term it is intermediate algebra and the plane trigonometry.

(c) There is a special class in each half of the twelfth year of mathematics. The first covers the trigonometry and some parts of the analytical geometry; the second covers the solid geometry and some elementary calculus.

(d) During this year a special class was conducted in algebra in which a unit of demonstrative geometry was introduced into the regular work. This class is now being continued as a separate group, and an attempt is being made to fuse the plane geometry with some parts of the solid geometry.

(e) An attempt is being made to introduce uniformity into the ratings recorded by teachers through the use standardized achievement tests in algebra, geometry and trigonometry. While the pupils in general are not classified according to ability, a prognosis test is used in algebra and in geometry at the beginning of each term, in order that the teachers may have something to guide them in their treatment of their pupils.

(f) All pupils who elect at least one year of mathematics beyond the plane geometry are grouped into separate classes and take intermediate algebra and the trigonometry fused into one course, with the examinations at the end of the year instead of at the end of each term. This is not a selected group based upon ability. It merely separates these pupils from those who wish to have only one half year of inter-

mediate algebra and nothing more. The course is presented, as far as possible, as a year of mathematics, and not as algebra and trigonometry. This course has been tried with two groups, each consisting of about 175 pupils.

Beginning with September, 1930, the pupils in algebra I and in geometry I will be grouped on the basis of the Orleans Prognosis Tests. The lowest quintile will be grouped into separate classes and be given a modified course, different from the academic course which leads to the New York Regent's examination, and which it is hoped will better suit the ability of the pupils.

5. *Franklin K. Lane High School,* Jacob Peshkin, Chairman.

(a) Repeaters in the first term of plane geometry are grouped in special classes where they are given the minimum syllabus of the second half of the plane geometry, while at the same time they are drilled in the work of the first term. The method is modified to meet the needs of the pupils. At the end of the term each of these classes is divided into three parts. Those who displayed more than average ability all term are permitted to take the Regent's Examination. Those in the second group are promoted into a Regent's class the following term. The pupils in the third group, constituting about twenty per cent of the repeater group or five per cent of the whole group are required to drop geometry, having failed twice in succession.

(b) For the normal classes in all grades, an honor scheme has been adopted by means of which it is hoped to provide for pupils of superior ability. As soon as possible after the beginning of each term, the better pupils of all classes are designated as "honor pupils." These must cover an honor syllabus in addition to the regular syllabus. This consists of one or two exercises to be done in addition to the regular assignment, more difficult exercises, puzzle problems, theorems and exercises not in the regular syllabus, the writing of papers on the history of mathematics and the biographies of famous mathematicians. The honor pupils present their exercises and papers for the benefit of the rest of the class. At the end of the term special honor examinations are held. Those who show power in mathematics as judged by the results of these examinations are awarded a "certificate of honor" in mathematics. A notation is made of this on the record sheet of each successful candidate for permanent reference.

6. *James Monroe High School,* Nathaniel Silberstein, Chairman.

(a) Pupils who have failed in geometry are grouped into separate "repeater" groups for special treatment. The same is done with pupils who have failed in geometry II.

(b) This year an experiment is being tried with four classes of a little over 100 pupils with a combined course in algebra and trigonometry like the one described under George Washington High School (f).

7. *Manual Training High School,* Harry Eisner, Chairman.

A non-Regents geometry course is given to about 40 per cent of the geometry pupils. These pupils are selected on the basis of their achievement in algebra.

8. *Morris High School,* William Gaylor, Chairman.

(a) The school is so organized that it has a school within the school. In it are all pupils of high average. These pupils recite in groups unmixed with those of lower grade. This gives the teacher of mathematics the opportunity to do more intensive work than is usually given to an ordinary group.

(b) Some calculus is taught in the advanced algebra class, and a little practical work with the transit is given to the classes in trigonometry.

9. *New Utrecht High School,* Meyer Weiner, Chairman.

Pupils who have failed in algebra I and in geometry I are grouped into special "repeaters" classes for special treatment. Those who do good work are permitted to enter the Regent's Examination.

10. *Stuyvesant High School,* William E. Breckenridge, Chairman.

(a) Use of the slide rule for checking in trigonometry.

(b) Course in surveying equivalent to the first course in any college.

(c) Special course in mathematics for gifted students (beginning September, 1930).

(d) Emphasis on checking and precision throughout algebra and all computation.

(e) Slow progress classes in algebra and geometry.

11. *Thomas Jefferson High School,* Arthur Haas, Chairman.

(a) All pupils are grouped in classes, as far as possible, in accordance with their grades of the previous term. Those who rank was above 80% are in "A" classes, those who failed are in an "F" group and the rest are called "B" pupils.

(b) There are two classes of commercial pupils, largely, juniors and seniors, who attempt to complete the first and second term algebra course in five months. Those who do good work during the first six weeks are grouped together and allowed to take the Regents Examination, subject to the 75% ruling. The rest are promoted to a regular second term algebra class.

(c) There is one class in the combined intermediate algebra and trigonometry course that is described under George Washington High School (f). This is an assorted group representing a cross-section of the A, B, F classification.

12. *Wadleigh High School,* John A. Swenson, Chairman.

The function concept and variation are emphasized throughout the mathematics course even in geometry. Differentiation and integration of algebraic polynomials are introduced into the work in advanced algebra which is given in the latter half of the eleventh year. Advanced algebra, trigonometry, and solid geometry have for the sake of state examination purposes their usual place in the program but the calculus is used as the fundamental tool in a great deal of this work.

National Council Slogan I

Every Teacher of Mathematics a Member of The National Council of Teachers of Mathematics

The Skills Involved in Problem Solving in Elementary School Arithmetic

By GUY A. WEST
Teachers College
Silver City, New Mexico

ACCURATE DIAGNOSIS DEPENDS upon adequate analysis. This is true of that part of arithmetic which deals with problem solving as well as of the fundamentals of the subject.

The fundamental skills of arithmetic have been analyzed by several authorities, but little has been accomplished toward setting up a reliable and complete analysis of the skills involved in problem solving. No one would argue that the teaching of problem solving is similar in principle to the teaching of the basic facts of arithmetic. On the other hand, few would doubt that a detailed analysis of the skills involved in the former would have considerable value for the teacher who finds that John Jones can not solve certain problems. His inability to do so remains the unsolved mystery to many teachers.

No teacher may know all the possibilities of her subject until she has before her an analysis of the factors involved therein. Analyses aid as a guide not only in introducing new processes, but also in maintaining skills in the various processes and in planning remedial work. Teachers have always known that pupils fails to solve some problems. Analyses, if put to proper use, may enable the teacher to go a step further and determine the reason for such failure or at least the particular point which baffles the pupil. Following this, it should not be difficult to take the necessary remedial step or steps.

Now, there have been many excellent studies which show that the cause of thirty to sixty per cent of the errors in problem work is lack of comprehension. There are at least a dozen books on the teaching of arithmetic which set down four or five "steps" that must be taken by the pupil who undertakes to solve a problem. The following is typical of such lists:

1. Grasp the conditions (Comprehension).
2. Separate into smaller units.
3. Plan the solution.

4. Solve in the best way.
5. Check the results.

Many studies have sought to throw more light on the many factors involved in comprehension. Some of these investigations report data on the language used in the problems; others are directed at the type of activity or situation involved, and at least one gives a detailed statement of the various sources of problem materials. Still others have attempted to show the relative social use of certain operations and combinations of operations in arithmetic.

In a recent study, the writer attempted to gather up certain of these data and combine them in usable form. An attempt was also made to determine the frequency and variety of use of the different forms of stating a problem. The purpose was to try to answer in detail the following questions:

1. Addition.
2. What forms of statement are used?
3. What is the arithmetical content?
4. What are the sources of problem materials?
5. How many steps or processes are involved and what combinations of processes?

Obviously, any problem is presented in one of three ways:

1. Orally.
2. In writing.
3. As an actual life situation.

Investigation revealed that there are several commonly used forms of statement. The results in the case of one text appear below:

Form of statement	*Frequency of use*
1. Simple question	28
2. Simple imperative	22
3. A statement of the facts plus question	230
4. A statement of the facts plus imperative	37
5. Conditional question	57
6. Conditional imperative	5
7. Two or more sentences giving the facts plus question	111
8. Two or more sentences giving the facts plus imperative	2
9. Several questions in series	6
10. Combinations of above forms	91

Closer investigation revealed that the simple question is used in a variety of forms. Most commonly we find questions of the type: How many are four apples and three apples? Yet there are numerous

questions stated in an inverted form: four and three are how many? It may astonish some to know that the first problem in a certain third-grade arithmetic text reads in this form.

In tabulating the skills under question three, the materials of the Denver Course of Study were taken as typical. Under question four, data from other studies were used.

In considering the number of processes involved, it was found that there are at least six types of one-step problems:

1. Addition.
2. Subtraction.
   (1) Comparison or difference type.
   (2) Take-away or remainder type.
   (3) Additive type.
3. Multiplication.
4. Division.
   (1) Measuring type.
   (2) Partitive type.
5. Reduction ascending.
6. Reduction descending.

Of two-step problems there are fifteen types including all possible combinations of the four operations. There are also sixty-four combinations of three-step problems, although the relative frequency of use of each is not known.

While it is impossible to go into detail, the rough outline given here will serve to indicate some of the possibilities of such analyses. In view of the facts presented in the original study it was concluded that the analyses are of use as:

1. A guide in the construction and organization of the course of study in arithmetic.
2. A guide for presenting the subject systematically in the classroom with respect to:
   a. the variety and frequency of exposure to the many factors involved in order that each may receive proper emphasis.
   b. the proper sequence of the various skills from the standpoint of economy of teaching.
3. A basis for diagnosing pupil difficulties.
4. A basis for the scientific construction of remedial work and for the organization of the proper type of remedial teaching.

A Mathematics Exhibit

By M. Cottell Gregory
Louisville Girls High School, Louisville, Ky.

Display boards and tables and a poster on which was printed in bold type "Mathematics Exhibit", aroused some curiosity among the students of The Louisville Girls High School last spring. They were accustomed to the exhibits of the art and home economics departments, but a mathematics exhibit! Who ever heard of such?

Mathematics to many students is dull and uninteresting, something to be endured with much labor in order to earn a few credits.

There are, of course, a few students who delight in a difficult task well done, and there are a few, inspired by a teacher who loves her subject, who catch a glimpse now and then of the real beauty of that subject whose laws are eternal, who see and appreciate mathematics in the world about them. But, alas! there are scores of students each year who manage to "get by" with a sigh of relief. It was for these students especially that the idea of a mathematics exhibit first presented itself. Some of the work was done in the classroom but most of it was supplementary work done outside of class time.

The algebra classes made graphs of various kinds. One statistical graph showed the average temperature in Louisville month by month for the past three years, the student obtaining the data from the weather bureau. A circular graph gave a student's own idea of the causes of failure in high school. A bar graph showed the relative enrollments of the freshman, sophomore, junior, and senior classes. Data collected from the daily papers were developed into graphs. Graphs as a means of solving equations and problems took on new life. Even the hyperbola, parabola, ellipse, and circle became interesting as pictures of algebraic equations.

The classes in geometry showed a great deal of ingenuity in the material which they contributed. Borders of original designs for linoleum and wall paper, designs of doors and windows, were colorful and attractive. One student interested in modern tendencies in art developed a human face in geometric figures.

Scrapbooks of all kinds created interest. From the first day of the term to the very last, attention was called to geometry in the world about us. Colored pictures from magazines were cut out and mounted to show geometry in architecture, industry, art, and nature. Seed and flower catalogs were searched for evidences of geometry in the botanical world. Even the bee and the spider seemed to possess an instinct for geometry. Geometric patterns in snowflakes and salt crystals brought a thrill. The quotation from Plato, "God eternally geometrizes," which was found when reading a bit of the history of geometry, took on new meaning.

"Geometry in the Home" was the title of one interesting book, in which the exterior and the interior of the home were carefully studied for evidences of geometry. Each room with its furnishings was found to abound in geometric patterns.

One very attractive scrapbook was called "The Geometric Vogue." The cover itself was quite unique and the pages revealed all sorts of things that lend charm to the feminine costume. There were bits of the new prints showing geometric patterns, pictures of costume jewelry, shoes, scarfs, and sweaters, with their interesting triangles, circles, and polygons. And then there was a geometric girl clad in a geometric sports costume.

Some students preferred themes to scrapbooks. There were brief sketches from the history of mathematics. "Geometry in Modern Art," "Mathematics and Music," and "Symmetry" were topics chosen for papers.

A poem called "A Dream" written and illustrated by one student told about a boy who while wearily trying to work his way through a problem fell asleep. As he traveled in his dream he discovered geometry everywhere. Finally he came to a cemetery and discovered a tombstone in true geometric design with his own name on it. He awoke with a start and resolved to learn more about geometry.

The class in solid geometry made paper weights of plaster and painted them bronze. Much time and patience were required to make the patterns of the polyhedrons out of sheet metal before the plaster models could be made. All of this was done at home. A solid geometry church attracted much attention with its hexagonal pyramid tower and triangular prism roof. Doors and windows with their arches were carefully constructed. Geometry in the houses

of primitive people was shown in drawings of the Indian tepee, the Eskimo igloo, and South Sea Islander's huts.

Note books showing classroom work well done received their share of praise.

Where is the educational value in all of this? I shall let three of the students answer.

"I never knew that mathematics could be so interesting."

"I have been walking through the world blindly."

"There actually seems to be a mathematical principle underlying everything."

Tartaglia - The Stammerer

NICOLO TARTAGLIA, whose portrait appears as the frontispiece in this issue, was born in Brescia and was one of the greatest Italian mathematicians of the 16th Century. A saber cut in the face which he received as a child when Brescia was stormed by Gaston de Foix (1512) resulted in an imperfection in his speech. This defect caused him to be given the nickname of Tartaglia ("the stammerer") "which name he formerly used in all his published works."*

Tartaglia was self-educated, but he managed to become proficient enough in mathematics to earn his living by teaching the science in several cities.

Tartaglia is given the credit for substantially completing the solution of the cubic equation. He afterwards told Cardan his secret and the latter published it in 1545 as his own.

Tartaglia is said to be the first to apply mathematics to artillery science. He also wrote what authorities consider the best arithmetic that appeared in Italy in the 16th Century. This arithmetic is a truly remarkable work especially for the time of Tartaglia. He also published (1543) editions of Euclid and Archimedes.

* Smith, David Eugene, *History of Mathematics*, Vol. I, page 297. Ginn and Company, 1923.

Mathematics Manuals and Keys

By HAROLD E. HUNTER
Southeastern Teachers College, Durant, Okla.

IT IS A COMMON PRACTICE for publishers of textbooks in mathematics, geometry texts in particular, to publish manuals or keys for the use of teachers of these subjects. Such manuals are usually characterized by two essential features: 1. the key proper, in which are found more or less complete solutions for all original exercises given in the text; 2. the manual, in which the authors give suggestions for the presentation of the subject matter of the text.

These manuals are published for the purpose of aiding busy or inexperienced teachers to teach their classes more efficiently, certainly a worthy motive. The question arises, however, as to whether or not the actual use of manuals tends to further this purpose. This article is written with the intention of presenting some observations concerning the advantages and disadvantages of manuals and keys in connection with the teaching of mathematics. The discussion is restricted to the field of junior high school series in general mathematics and senior high school courses in plane and solid geometry because the writer has these particular subjects under his direct supervision; but it would seem that the general conclusions drawn from a study of this restricted field might apply equally well to other grades in mathematics and even to other subjects in which the use of manuals is prevalent.

I must confess that I have at hand no statistics to show to what extent teachers of mathematics are using manuals and keys as an aid to their teaching, but I have found that beginning teachers evince a lively interest in a discussion concerning the propriety of using them. In fact, my attention was first focused on this problem by the inquiries of practice teachers in high school mathematics, and my interest was still further aroused by contact with salesmen of new texts in this field. One such salesman based the greater part of his sales talk on the excellent type of manual which accompanied the text in question. Another asked my advice as to the advisability of publishing a manual and key to accompany the text which he

advocated. As a result of the interest shown by these inquiries, I have reached the conclusion that the problem is of sufficient importance to warrant this discussion.

My first acquaintance with a textbook key was made during my high school days. Although I had no direct access to the key in question, due to parental intervention, some of my classmates in Caesar had no conscientious qualms concerning their use of a "Pony" in preparing their lessons. I do not recall ever having seen a high school student using a mathematics key but I assume that a few students are able to procure them. Fortunately the publishers of all types of keys take every possible precaution to prevent their indiscriminate use by students. Hence in this article we are not concerned with the illegitimate use of keys by students, but, on the contrary, we are vitally interested in the use, or mis-use, of keys by teachers.

Let us now consider the circumstances under which the key is to be used. As stated above, it is published for the benefit of busy and inexperienced teachers. Presumably, then, some teachers are too busy to teach geometry, let us say, without the aid of a key. But does the use of the key appreciably lessen the time which must be spent either in preparation or in the correction of papers? Possibly it does, but we must remember that the average exercise in geometry can be solved in a number of different ways and that the key usually gives only one solution. Would this busy teacher take time to study each individual solution carefully and give credit where credit is due, or would he consider that any solution other than that given in the key is incorrect and thus do injustice to the student who has developed a different type of solution? It would seen that, in the former case, very little time is saved and, in the latter, the saving of time is a positive detriment to the efficiency of the teacher. Apparently, as a time saver the key has a very insignificant place in the classroom. We must admit, however, that the manual, in which suggestions for the presentation of the subject matter are given, might be the means of saving time for the busy teacher, as well as offering valuable aid along other lines. This phase of the matter will be considered at greater length later.

As for the inexperienced teacher, we believe that the same general observations will hold good, if we assume that this teacher, though

inexperienced, is well prepared in both subject matter and methods of teaching that particular subject. But should we make this assumption? In many cases we would not be justified in so doing. We come, then, to the real reason, or excuse, for the existence of the key proper and probably that of the manual as well. We are expecting people to teach mathematics in our high schools who are not thoroughly grounded in the fundamentals of the subject. Such teachers must either make use of a key or admit to their students their inability to work original exercises. Our conclusion is that the key is of no particular benefit to the busy or inexperienced teacher, but that its only excuse for existence lies in the help rendered to the unprepared teacher. What is the effect on the pupil?

A few years ago I had as a student in my college class in solid geometry a young man who had been teaching this subject for two years in a high school in a nearby town. He admittedly enrolled in the course for the sole purpose of securing the credit, and not with the idea of enhancing his ability as a teacher. He stated that he could solve any original exercise in the text, and he could (with the aid of his key). However, he was greatly surprised to find that I considered it possible to prove a proposition in more than one way. After I had demonstrated the truth of the statement by several examples, he asked me how it was possible to tell when a proof was correct in case it happened to be different from that in the text or key. My answer was an attempt, which covered several weeks, to teach him the meaning of logic as a criterion of final proof of any statement. I believe that I met with partial success, but I am also sure that he was never absolutely convinced that a proposition had been proved or an exercise solved unless the key corroborated the proof or solution. To think that such conditions exist in our profession! To think that, knowing they do exist, we do so little to correct them! Do little? Possibly I am too pessimestic but I repeat that the very existence of keys and manuals for our texts is mute testimony to the fact that some of our teachers are not prepared as they should be.

Unfortunately I was unable to determine by direct observation the type of geometry student which this particular teacher had developed during his two years of experimentation, but I did gather sufficient indirect evidence to warrant the following conclusions:

1. His pupils gained a fairly comprehensive idea of the facts of solid geometry, i.e. they knew the principal definitions, formulas, etc.

2. He had given sufficient drill on the proofs of propositions to enable the class to repeat them with some degree of skill, i.e. they had gained a mechanical knowledge of the principal propositions.

3. He had given them no training in analysis or independent thinking.

Thus one very important, if not the most important, element in the teaching of geometry had been neglected. Had his students derived from that course value commensurate with the time spent? I doubt it. I cannot believe that this type of teaching is worth while, and yet the teacher who depends upon a key can hardly expect to rise above the level here depicted.

Let us now consider the manual as distinct from the key proper. Teaching suggestions are always valuable. It is true that the teacher should have a thorough grounding in the methods of teaching before attempting to teach a class, but let us suppose that the author has introduced some new device for explaining some difficult principle. Assuredly he should give a complete explanation of the use and value of this device. He should also give the teacher some idea as to the relative importance of the various propositions and the different chapters, as the text may have to serve one teacher for a maximum course whereas another has only sufficient time to cover the bare essentials. Such suggestions are frequently found in the manuals which are in current use and might be considered as furnishing sufficient justification for the existence of the manuals. But why must these suggestions be included in a separate book? Why not make them an integral part of the text itself? We find that this is being done in many of our more recent texts. We find these books including more and more teaching suggestions as related to old material and new devices for explaining that material. It would seem that the modern author of mathematics texts had obviated the necessity for a separate manual. Again and again we reach the conclusion that the manual and key is of value only to the teacher who does not deserve to be so named, rather let us say to the counterfeit teacher, he who is attempting to do that for which he is not prepared.

The discussion thus far has been based almost entirely on the

study of keys to texts in geometry. Algebra has not been considered for the reason that very few authors deem a manual or key necessary for books on this subject. Answers are usually given either in the text or in a separate answer book but it is considered unnecessary to outline the method of solution. This idea is based partially on the fact that exercises in algebra can be definitely classified as belonging to a given type for which the method of solution is known, whereas exercises in geometry cannot be so classified. Furthermore we find that most of our high school teachers have had college training in algebra while very few have had any training in geometry beyond that received in high school. Probably to this fact, more than any other, is due the condition noted above, i.e. that we have many poorly prepared teachers of geometry and consequently have published manuals and keys for their benefit.

We have reached the conclusion that in an ideal situation manuals and keys would be superfluous but we are also agreed that we are far from this ideal. How can we improve conditions and thus come a step nearer to our ideal? In the first place, a great responsibility is placed on the shoulders of our principals, superintendents and school boards. Not only must they use great discrimination in the selection of teachers but they must do everything in their power to arrange their curricula so as to enable their teachers to handle only those subjects in which they are fully prepared. In the second place our colleges, teachers' colleges especially, must offer advanced training in geometry as well as in algebra and must include this advanced training under the list of requirements for mathematics teachers. Many colleges have taken this step in recent years but the number is all too few and the great need for this work has not been fully realized. May the time soon come when no teacher will be asked to teach, or endeavor to teach, any subject to which he cannot do full justice! When that time comes we will have no further use for manuals and keys to our texts.

In conclusion, may I say that this article has been inspired by my sincere belief that a real evil exists, not in the mere fact that we have manuals and keys, but rather in the fact that there is a need for them, and an earnest desire to do my small part in correcting it.

NEWS NOTES

William S. Schlauch, the new vice-president of the National Council of Teachers of Mathematics and for twelve years Chairman of the mathematics department of the High School of Commerce in New York City and for twenty-four years a member of the faculty in that institution was recently elected to the position of assistant professor of mathematics at New York University.

Mr. Schlauch was showered with gifts and testimonials. A gold watch was given him by the faculty as a personal token of affection and esteem. He also received a set of the new edition of the Encyclopedia Britannica. A scholarship to be presented in his name to the member of the graduating class who excels in mathematics and stands high in service was established as a memorial prize to be given annually.

Members of the faculty were sincere in their praise and regret.

"I do not know how we shall ever replace Mr. Schlauch." Principal Edward J. McNamara said, "His leaving creates a hole that will be hard to fill."

Professor Oscar Lynn Kelso, one of the pioneers of the educational movement in Indiana and for 30 years head of the mathematics department of the Indiana State Normal School, died at the Union Hospital on July 13 after a lingering illness of more than two- and one-half years. Professor Kelso was 75 years of age at the time of his death and the greater part of that long period had been spent in educational service.

Professor Kelso was the author of the "Kelso Arithmetic for High School and College" and collaborated with Robert J. Aley, president of Butler College, in revising the Cook-Cropsey arithmetic for use in the schools of Indiana. As a student at Indiana University, he was a member of the Phi Delta Theta fraternity. He was also a member of the State Teachers Association, the State Academy of Science, and the Terre Haute Science Club. He was also one of the pioneer business men of Terre Haute.

Not only has Indiana State Teachers College lost one of its staunchest friends but education throughout the entire state will feel his loss.

The mathematics association annual spring meeting was held at the University of California, April 26, 1930.

Forenoon Session

"Articulation of Junior and Senior High School Mathematics," Miss Aurora Bettencourt, Woodrow Wilson Junior High, San Jose.

"Correlation between Junior and Senior High School Mathematics," Miss Elizabeth Smith, Horace Mann Junior High, San Francisco.

"Results of Homogeneous Grouping and Comparison Tests in Mathematics Classes of Balboa High School, San

Francisco," Miss Irene Pauly, Balboa High, San Francisco.

"Algebra Prerequisite for High School Chemistry," Mr. A. Schwartz, Lowell High, San Francisco.

"Recent Contributions of Research to the Teaching of Mathematics in Secondary School," Miss Violet Lowe, Oakland High.

Business Meeting

Afternoon Session, 2:00

"Junior High School Mathematics,—What Is It? Can It appear without Certain Changes in the Senior High School Course?" Miss Edith L. Mossman, Garfield Junior High, Berkeley.

"The Proposed One Year Course in Plane and Solid Geometry," Round Table, Leader: Mr. A. D. King, Polytechnic High, San Francisco. Discussion: Mr. D. T. McCarthy, Galileo High, San Francisco.

"Tests and Measurements," Round Table, Leader: Miss Maud Oakes, Balboa High, San Francisco. Discussion; Miss Hennessy, Commerce High, San Francisco.

Reception Committee: Miss Tuttle, Mr. Yulich, Mr. Albrecht (E. J.)

Luncheon Program Committee: Miss Giddings, Frick Junior High, Oakland.

Decoration Commitee: Miss Alma Adams, Roosevelt High, Oakland.

Exhibit Committee: Miss Constance B. Abbott, Berkeley High; Mrs. Helen T. Hoefer, Richmond High.

President: Mr. A. L. McCarty, Lowell High School, San Francisco.

Secretary: Miss Adelaide Bartlett, Everett Junior High School, San Francisco.

The last meeting of the Detroit Mathematics Club for this school year was held Thursday, May 15, at Sherrard Intermediate School.

The following program was given:

1. "If," Miss Lenore Herz Sherrard Intermediate.
2. "Individualization in the Seventh and Eighth Grades," Mrs. Fry, Sherril School.
3. "Making the Most of the Formula in Geometry," Mr. C. E. Smith, Redford High School.
4. "Summary of the Club's Work in Curriculum Revision," Miss Ruth Utley, President.
5. Election of Officers.

The Annual Meeting of the Mathematical Association was held at the London Day Training College, Southampton Row, London, W.C. 1, on Monday, 6th January, 1930, at 2:30 P.M., and on Tuesday, 7th January, 1930, at 10 A.M. and 2:30 P.M.

Monday Afternoon, 6th January, 1930

1. 2:30 P.M. Business:—

(a) The Report of the Council for the year 1929.

(b) The Treasurer's Statement for the year 1929.

(c) The Election of a President for the year 1930.

The Council nominated Professor A. S. Eddington, M.A., F.R.S., for the office of President.

(d) The Election of Officers and Council for the year 1930.

Miss M. J. Griffith and Mr. F. G. Hall retire from the Council, and are not eligible for re-election. Miss R. H. King (Head Mistress of the County High School for Girls, Colchester) and Mr. C. J. A. Trimble (Christ's Hospital, Horsham) have been nominated for Election in their places.

(e) Consideration of a special report by the Council on the relation of the Branches to the Association. If

the proposals contained in this report are approved, alterations must be made in some of the Rules of the Association and the proposals will be submitted to a special General Meeting to be summoned for that purpose.

Monday Afternoon, 6th January, 1930.

2. 4:00 P.M. Discussion: "Arithmetic of Citizenship"; opened by B. L. GIMSON, B.Sc. (Bedales School).

3. 5:00 P.M. Interval.

4. 5:30 P.M. Lecture: "The use of Spherical Harmonic Functions in Mathematical Physics", by Professor S. CHAPMAN, M.A., D.Sc., F.R.S. (Imperial College of Science and Technology).

Tuesday Morning, 7th January, 1930.

5. 10:00 A.M. Discussion: "Problems of Individual Education, with special reference to work in Mathematics"; opened by G. W. SPRIGGS, M.Sc., F.C.P. (The Tiffin Boys' Secondary School and the Technical College, Kingston-upon-Thames).

A brief survey of the principles and practice of the Dalton Plan, and an estimate of these in the light of practical experience. Some objections and difficulties, and the steps taken to remove them. The new laboratory equipment and the development of a technique to deal adequately with the problems in the Education of the Individual. Details of the application of the scheme to Mathematics. Fundamental principles of the new practice.

6. 11:30 A.M. Interval.

7. 11:45 A.M. Lecture: "Gunnery and some of its Mathematical Problems", by Professor W. M. ROBERTS, M.A. (Royal Military Academy, Woolwich).

A little history. Some very elementary applications of Trigonometry. Air resistance and its difficulties. Probability.

Tuesday Afternoon, 7th January, 1930.

8. 2:30 P.M. The President: "Mathematics for study of Frequency Statistics".

(A) Study of a new subject may lead to new mathematical problems or to new uses of existing methods. (B) Treatment of observed facts differs from treatment of mathematical problems in two respects. (1) Facts only given for limited number of cases: leads to interpolation, etc. *Central differences.* (2) Observations only approximate: leads to theory of error. (C) "Gaussian" formula for frequency of error. Extension to frequency-distribution of single variable. Moments. Quadrature. *Euler-Maclaurin formula.* (D) Correlation of two or more variables. Frequency of joint occurrence. Two special classes of cases: (1) correlation of coexistent attributes, (2) correlation of errors of sampling. *Elements of tensor notation.*

9. 3:30 P.M. Interval.

10. 3:45 P.M. Discussion: "The Mathematician in Ordinary Intercourse"; opened by Miss HILDA P. HUDSON, O.B.E., Sc.D.

A mathematical reputation is a barrier to intercourse and tends to loneliness. Handicap and advantages due to our training; our unintelligible language; habits of mental arithmetic, of stressing the weakest link and the extreme case, of *reductio ad absurdum.* Certain services we can render easily, certain temptations we escape. We ask too much of the world, but we safeguard its standards of accuracy in thought and terminology, of laboriousness, of evidence, of truth.

NOTE. (i). Tea was provided, at 9d. per head, in the interval on Monday

and at the end of the meeting on Tuesday; and the rooms, including those containing the Publishers' Exhibition, remained open on Tuesday until 7 P.M.

(ii). A Publishers' Exhibition was arranged in connection with the meeting and was open on both days.

(iii). Members who wished to take part in any of the discussions were invited to send their names to one of the Honorary Secretaries before the meeting.

C. PENDLEBURY, *M. A.*,
39 Burlington Road,
Chiswick, London, W. 4.
MARGARET PUNNETT, B.A.,
The London Day Training College,
Southampton Row, London, W.C. 1.

TO THE EDITOR OF THE MATHEMATICS TEACHER:

The following announcement of a new educational periodical, the "Junior College Journal," is sent you in the hope that you may be able to make some mention of it in your columns. Your courtesy in this will be greatly appreciated.

Sincerely yours,
WALTER CROSBY EELLS, *Editor*

A new national educational periodical, the *Junior College Journal*, will begin publication in October, 1930. It will be published by Stanford University Press, and will be under the joint editorial control of the American Association of Junior Colleges and the School of Education of Stanford University. The new journal will appear monthly with the exception of the summer months.

Although there are more than four hundred junior colleges in the United States, with an enrollment of approximately sixty thousand students, there has been no periodical devoted especially to their interests. It is felt, therefore, that the new journal will fill a place not occupied by any other educational journal and should have an important influence in the unification and progress of the junior college movement in American education.

Dr. Walter C. Eells, of the Stanford University School of Education, will be editor-in-chief of the new periodical, with Doak S. Campbell, of Nashville, Tennessee secretary of the American Association of Junior Colleges, as associate editor. A national editorial advisory board of twenty men will include the members of the executive committee of the American Association of Junior Colleges, and other men who are recognized as national leaders in the organization and development of the junior college movement.

The national advisory board will consist of the following: J. Wynne Barton, Ward-Belmont School, Nashville, Tennessee; William John Cooper, United States Commissioner of Education, Washington, D.C.; H. A. Cross, Phoenix Junior College, Arizona; Frederick Eby, University of Texas, Austin, Texas; Mrs. Marion Coats Graves, Bronxville, New York; John Paul Jones, Junior College, Jackson, Michigan; William W. Kemp, University of California, Berkeley, California; Leonard V. Koos, University of Chicago; Edgar G. Lee, Christian College, Columbia, Missouri; J. B. Lillard, Sacramento Junior College, California; F. P. Obrien, University of Kansas, Lawrence Kansas; William M. Proctor, Stanford University; Nicholas Ricciardi, State Department of Education, Sacramento, California; Henry Suzzallo, Carnegie Foundation, New York City; Warren W. Way, St. Mary's

College, Raleigh, North Carolina; Frederick J. Weersing, University of Southern California, Los Angeles; Frederick L. Whitney, Colorado State Teachers College, Greeley, Colorado; James M. Wood, Stephens Junior College, Columbia, Missouri; and George F. Zook, University of Akron, Ohio.

The Association of Mathematics Teachers of New Jersey held their 37th regular meeting at Princeton University on March 1st, 1930. The following program was given:

Morning Session

Richard's Paradox, Dr. Alonzo Church, Princeton Universtty.

Fundamental Definitions in Trigonometry, Professor A. E. Meder, Jr., New Jersey College for Women.

Afternoon Session

Some graphical aids to the teaching of Mathematics in Colleges and Secondary Schools, Dean Allen R. Cullimore, Newark College of Engineering.

Secondary School Mathematics, Accumulating Information Regarding Learning and Teaching, Dr. J. Albert Blackburn, New Jersey College for Women.

Officers of the Society

Frederic J. Crehan, *President,* Columbia High School, South Orange, N.J.

Dr. Emory P. Starke, *Vice-President* Rutgers University.

Andrew S. Hegeman, *Secretary-Treasurer,* Central High School, Newark, N.J.

Council Members

The Officers and Professor Charles O. Gunther, Professor Richard Morris, Harrison E. Webb, Virgil S. Mallory, James B. Hawley, Miss Edna C. Burritt, J. W. Colliton.

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NEW BOOKS

Poetry and Mathematics. By Scott Buchanan (John Day Co. $2.50)

I wish that I could persuade every thoughtful teacher of mathematics to read this little book. I can promise fun, stimulating new ideas, and a delightfully Alice-in-Wonderland atmosphere. Poetry and Mathematics are treated as "of equal importance," as "two very successful attempts to deal with ideas."

Not quite so flattering to us are the comments that "a great many sometime students of mathematics try to persuade themselves that they haven't mathematical minds, when as a matter of fact they have only had non-mathematical teachers. Mathematics is not what most teachers of mathematics teach."

And the discussion goes on through equations, through functions, into Greek tragedy.

HARRY C. BARBER

The Aims of Education. By Alfred North Whitehead (Macmillan $2.50)

Some of Whitehead's books are too profound to be easy reading. Hocking says of his recent Process and Reality, for example, that it is "inspired but difficult." Not so with The Aims of Education (except in the last two chapters which one may be allowed to omit). It is full of phrases to be carried into lesson-planning and into the class room. In it the great philosopher and mathematician speaks in simple terms. To quote:

"The whole book is a protest against —inert ideas."

"From the very beginning of education the pupil should experience the joy of discovery."

"The curves of history are more vivid and more informing than the dry catalogues of names and dates—."

"The appreciation of the structure of ideas is that side of a cultured mind which can only grow under the influence of a special study."

"—algebra is the intellectual instrument which has been created for rendering clear the quantative aspects of the world."

Isn't that enough to whet your appetite?

HARRY C. BARBER

The Aryabhatiya of Aryabhata. Translated with notes by Walter Eugene Clark. Chicago, 1930. Pp. xxx + 90. Price $2.50

Aryabhata was a Hindu mathematician who flourished about 500 A.D. He wrote a brief treatise now known as the *Aryabhatiya.* It consists of four parts: (1) Ten stanzas relating to the revolutions of the sun, moon, and planets, and to the measures of time and space; (2) On Mathematics; (3) Reckoning of time; (4) The Sphere. For the purposes of this review the technical Sanskrit terms may be omitted.

The work is the oldest Hindu treatise on mathematics that has come down to us in what appears to be approximately its original form. What is apparently a predecessor, the *Suryasiddhanta,* a work devoted to astronomy, was written a century earlier,

but portions of it seem to have been added after Aryabhata's time. The second part of the *Aryabhatiya,* that dealing with mathematics, has already been translated into French (by Rodet in 1879) and English (by Kaye in 1908). and the whole work has recently been translated into English by Professor Prabodhchandra Sengupta (1927), this version appearing after Professor Clark's translation was made and too late for use in the printed text. These facts do not lessen the value of the work, which lies even more in the explanatory notes than in the translation itself.

For teachers of mathematics the chief significance of this edition lies in the text and the relatively few mathematical notes, whereas for the philologist the explanatory notes in his field, which take up most of the space, are the more important. For the general reader of scholarly tastes the most significant thing is the work as a whole, the fact that it has been done, and the still more important statement which Professor Clark has made, that "the present translation, with its brief notes, makes no pretense at completeness. It is a preliminary study based on inadequate material." This is a frank confession and is manifestly a true one. One of the first questions that a reader may naturally ask concerns the validity of the text, and the above statement is intended to state in advance that this has still to be settled—and similarly as to various details of less importance.

When we face the question of the contributions of any oriental race to mathematics, we are confronted by the same problem. We do not know, with anything like the degree of certainty that we know (let us say) the text of Euclid, the original text of any of the Chinese or Hindu classics. Textual criticism, in the European sense of the term, is only just beginning to be valued in the orient. We do not know when the *I-king* was written, how many authors have added to it, or when these additions were made. Similarly we do not know the original text of any other prechristian classic of the East. Even referring to a period as late as the 13th century, when the Chinese seem to have done some notable work in algebra, we are by no means sure of the validity of the three texts involved. It is in the work of scholars like Professor Clark that we may hope for light. He may easily be the head of a movement in this country that shall reveal to us much that is at present hidden with respect to the mathematics of the orient.

We may wish that he had, in this version, specified the source from which he translated, or had given more detailed explanations of certain Sanskrit terms; but when we recall his confession that this is but a first step, we should be glad that the step has been taken, hoping that he or some of his pupils may undertake anew the translations of Brahmagupta and Braskara, or may even attempt the more difficult task of translating Varahamihira, and all this with the needs of the mathematician in mind quite as much as those of the philologist.

A comparison of the translation of Professor Sengupta with that of Professor Clark may be of interest. The following are from the fourth part, "Gola or the Sphere."

Sengupta

22. Let an observer make a light globe of wood perfectly spherical and of even weight all round and let him turn it round in his selected unit of time through his intelligence by means of mercury, oil and water.

Clark

22. A light wooden sphere should be made round, and of equal weight in every part. By ingenuity one should cause it to revolve so as to keep pace with the progress of time by means of quick-silver, oil, or water.

It would seem that the former is the more literal translation, and the latter the more polished. I should like to know whether or not the next-to-the-last word ("and," "or") is actually in the best of the early texts. It makes considerable difference which is used—whether the line refers to a simple water clock, or a more complex instrument.

The notes given by Professor Clark are generous, but on the mathematical side those of Professor Sengupta are the more complete.

Although we still live in a time when the prices of scholarly books are such as to put these works out of the reach of most scholars, it is devoutly to be hoped that we can follow the plans of the French and issue books of this nature at a price which is not prohibitive. To pay $2.50 for 90 pages of text will seem a great hardship to a large number who might wish to add the book to their collections. Why should we not have popular editions, bound in paper, of books worth reading? Perhaps the answer is that nobody would buy them, but I doubt its correctness.

DAVID EUGENE SMITH